

# Expressivity and Complexity of Dependency Grammars

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**head**

loves

John

Mary

**dependent**

# Dependencies

# Current interest in DG

- \* **statistical parsing**

Eisner 1996; Collins 1997

- \* **languages with free word order**

Pl tek et al. 2001

- \* **syntax-semantics interface**

Debusmann et al. 2001

# The DG diversity

- \* **grammatical paradigm**
  - \* rule-based (Gaifman, Dikovsky)
  - \* constraint-based (CDG, XDG)
- \* **structural assumptions**
  - \* projective (Gaifman, Eisner)
  - \* non-projective (CDG, Nasr, XDG)

# Milestones

- \* **Gaifman 1965:**  
Projective dependency grammars and lexicalised context-free grammars are strongly equivalent.
- \* **Neuhaus & Bröker 1997:**  
The general word problem for unrestricted dependency grammars is NP-complete.

# What this talk is about

- \* **formal and computational aspects of dependency grammar**
- \* **language-theoretic expressivity**
- \* **complexity of recognition & parsing**
- \* **intermediate report on ongoing work for my PhD thesis**

# Questions asked

- \* How does DG relate to grammar formalisms other than CFG?
- \* How could a general framework look like, in which existing DG formalisms can be studied and compared?
- \* How can parsing techniques for other frameworks be transferred to DG?

# Focus of this talk

- \* **dependency structures**
  - \* structural constraints
  - \* non-projectivity
- \* **dependency languages**
  - \* mild context-sensitivity
  - \* extensional perspective

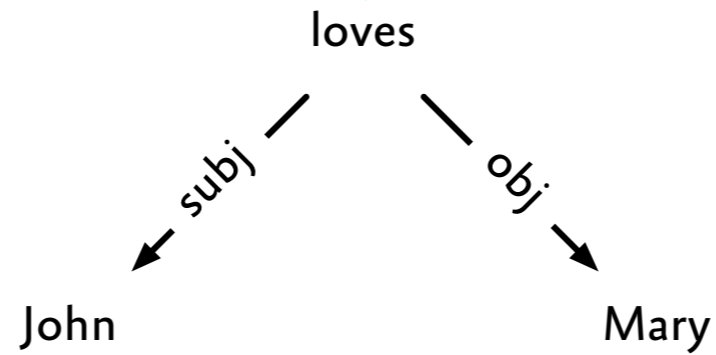


# Structure

- \* Introduction
- \* **Dependency structures**
- \* Dependency languages
- \* Complexity
- \* Conclusion & Future Work

# Dependency structures

head



dependent

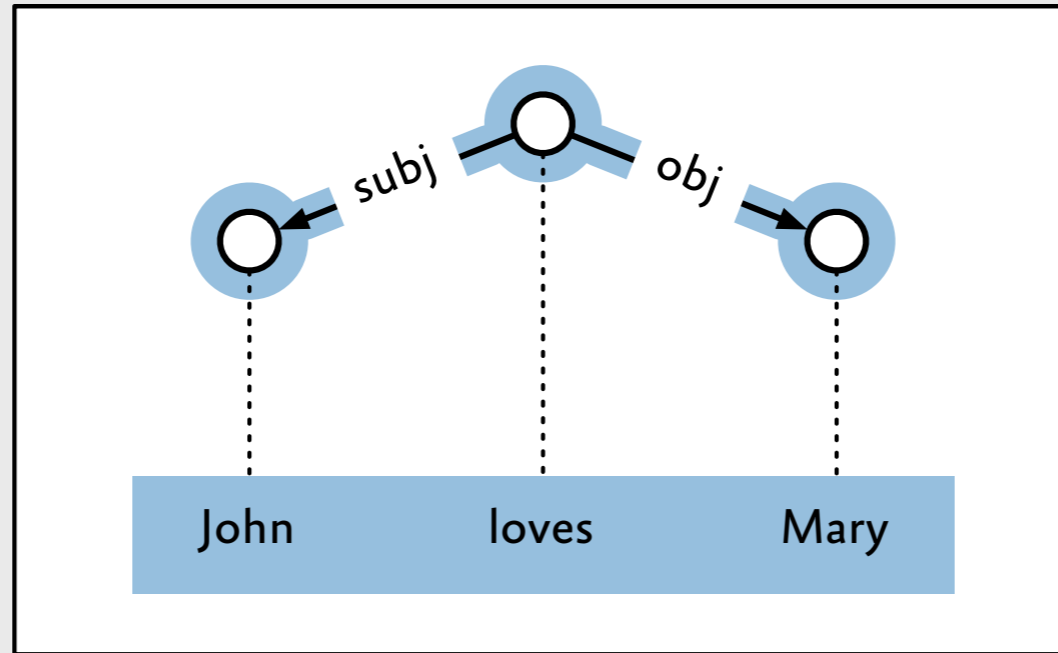
# Dependencies

# Common constraints (1)

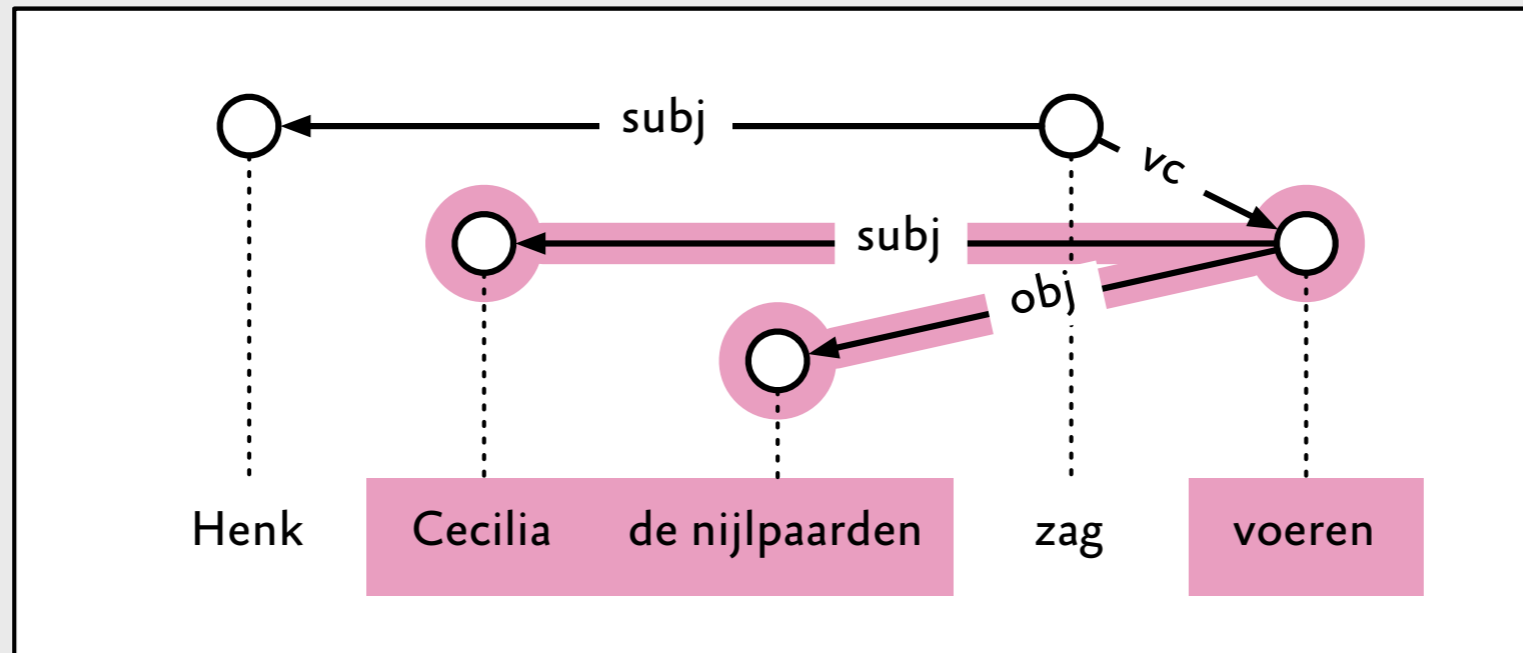
- \* **acyclicity:**  
no word depends on itself
- \* **indegree at most 1:**  
each word has at most one head
- \* **single root:**  
exactly one word without a head

# Common constraints (2)

- \* **projectivity:**  
the reflexive-transitive dependents of a word form a contiguous substring of the full sentence
- \* controversial; must be abandoned for languages with freer word order

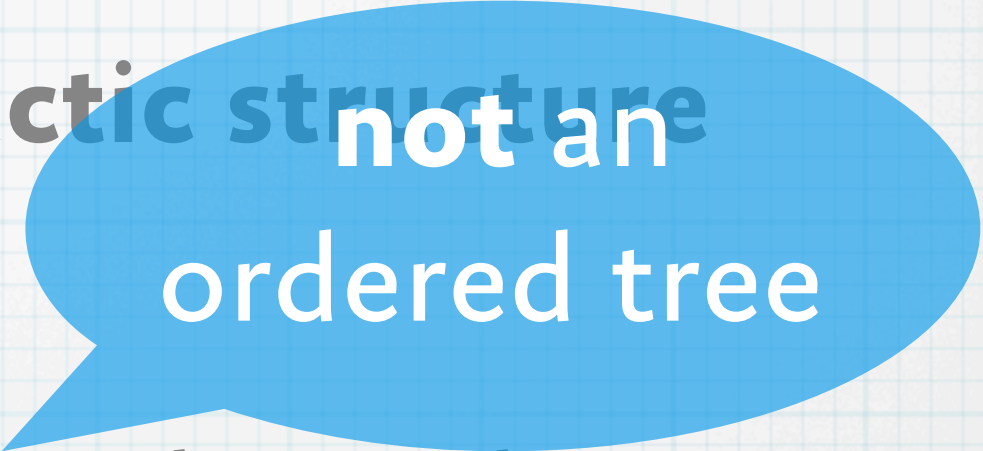


# Projectivity

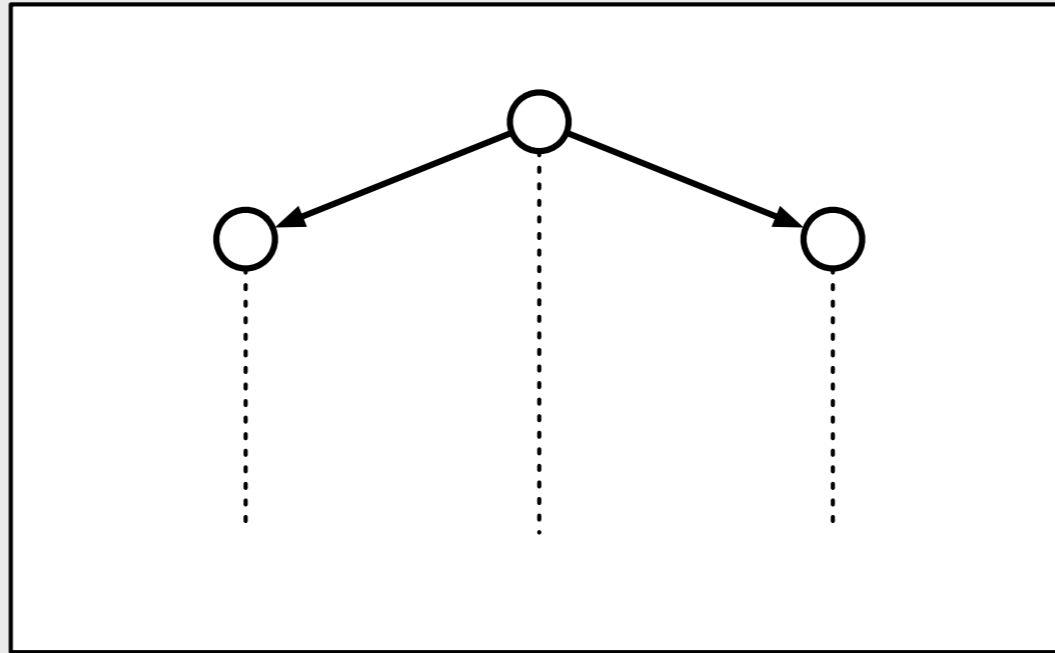


# Crossing edges

# Drawings

- \* **simple models of syntactic structure**
  - \* relational structure
  - \* forest + linear order on the nodes
  - \* **dimensions of non-projectivity**
    - \* quantitative aspect
    - \* qualitative aspect
- 
- not an ordered tree

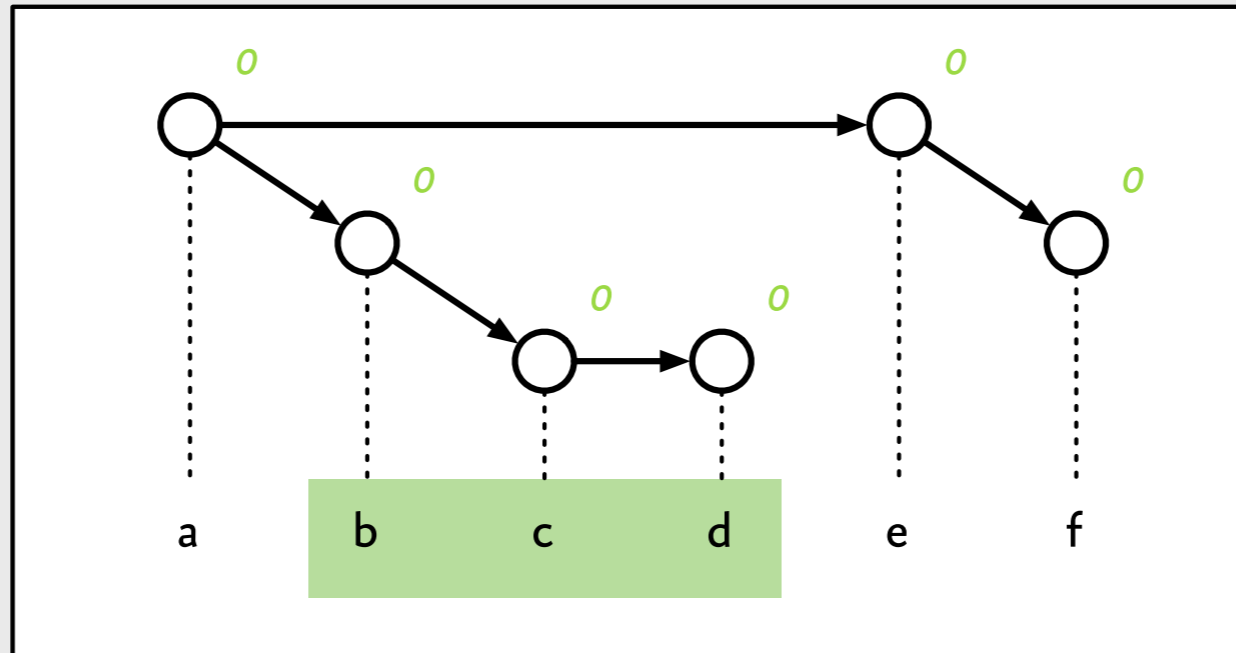




# Drawings

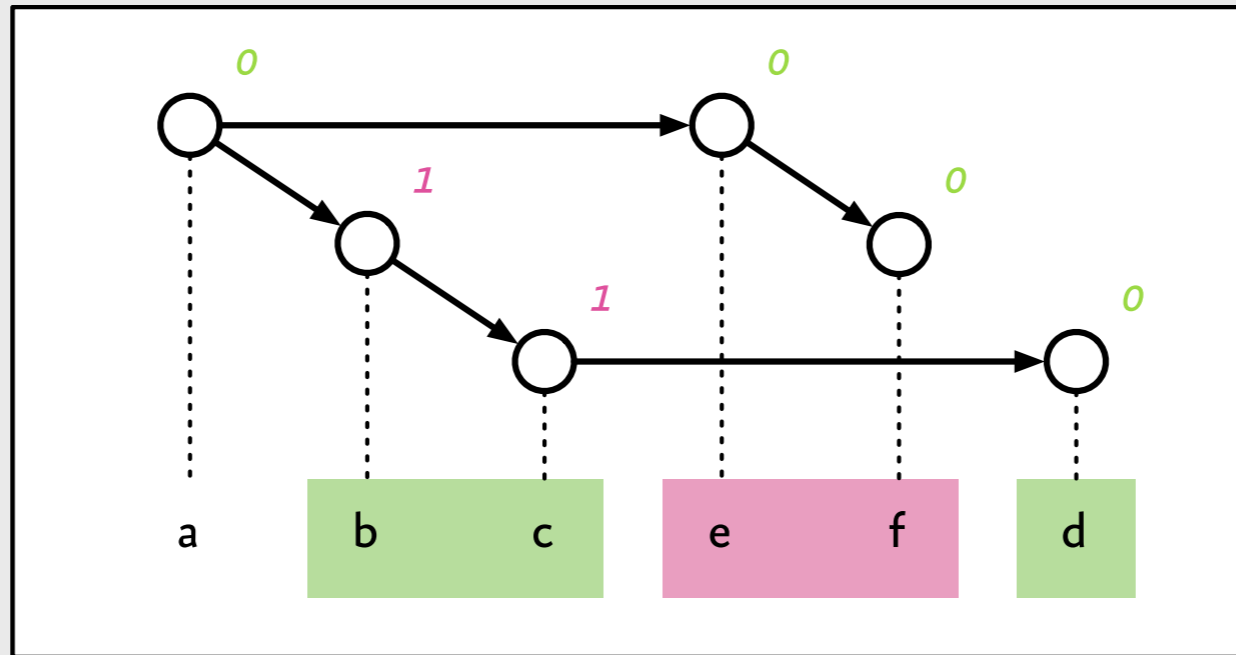
# Gap degree

- \* **gap degree of a node:**  
number of interruptions  
in the projection of that node
- \* **gap degree of a drawing:**  
maximum over the gap degrees  
of the nodes in the drawing

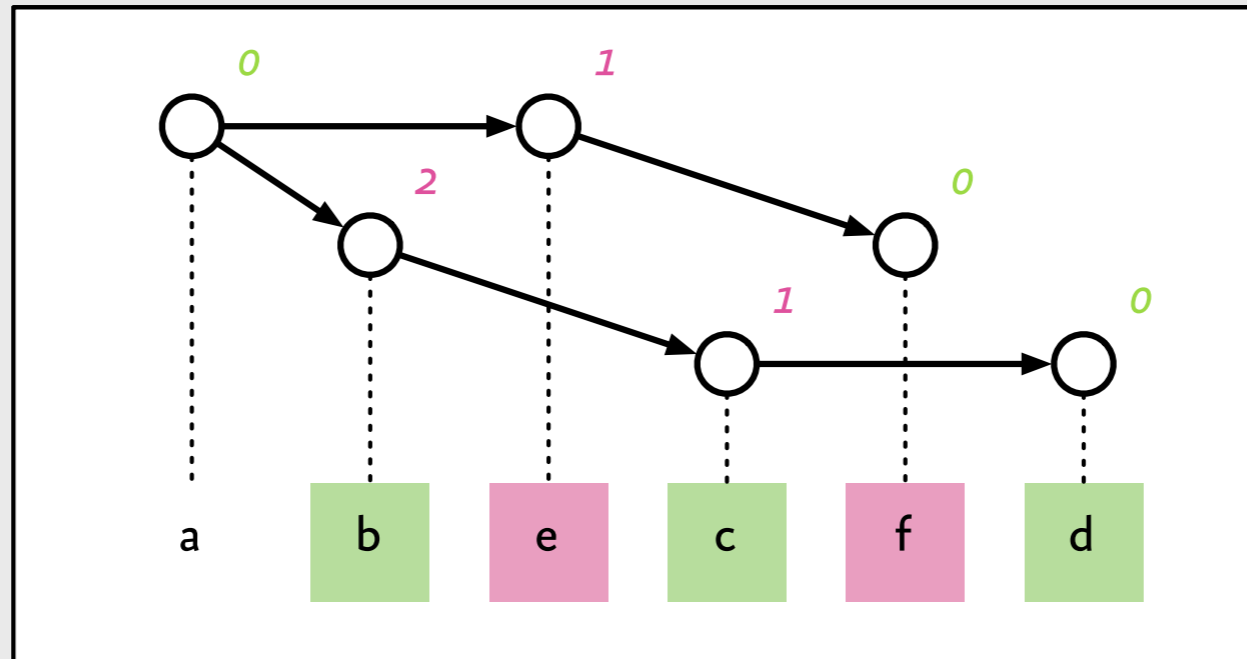


projective

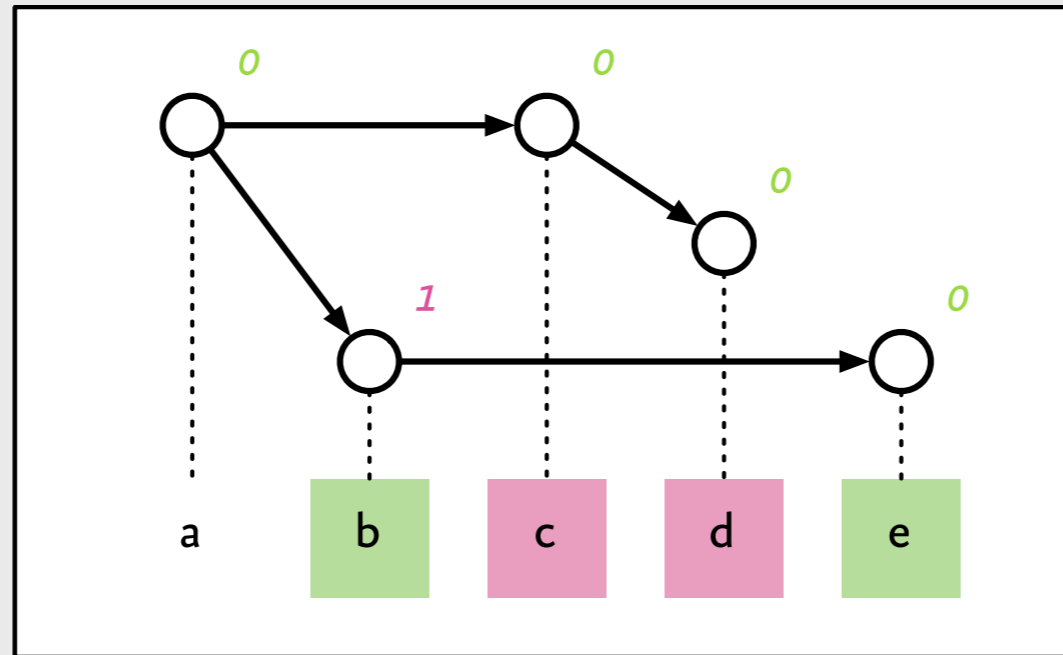
Gap degree 0



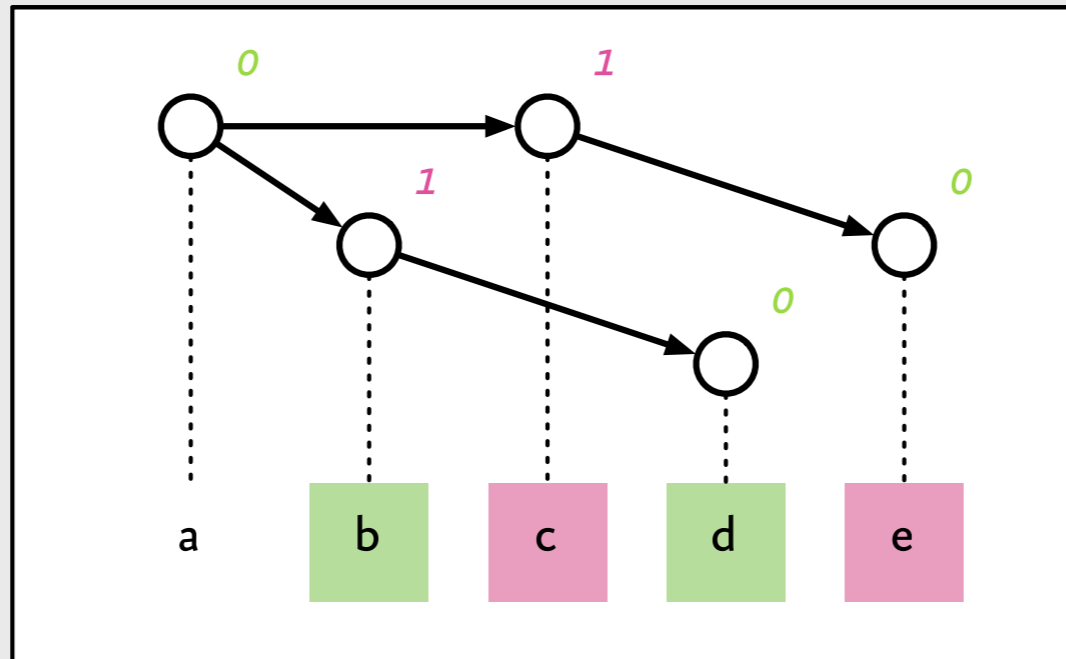
**Gap degree 1**



Gap degree 2



**Well-nested  
drawings (1)**



**Well-nested  
drawings (2)**

# Results

- \* The derivations of Lexicalised Tree Adjoining Grammar (LTAG) can be interpreted as drawings in a natural way.
- \* **The drawings induced by LTAG are well-nested and have a gap degree of at most 1.**



# Summary

- \* I propose **drawings** as a simple class of models for dependency structure.
- \* Drawings allow us to formalise and reason about various forms of **non-projectivity**.

# Structure

- \* Introduction
- \* Dependency structures
- \* **Dependency languages**
- \* Complexity
- \* Conclusion & Future Work

# Dependency languages

# Definitions

- \* **dependency language:**  
set of dependency structures
- \* **string language:**  
image of a dependency language  
under projections
- \* **dependency grammar:**  
specifies a dependency language

# CF languages

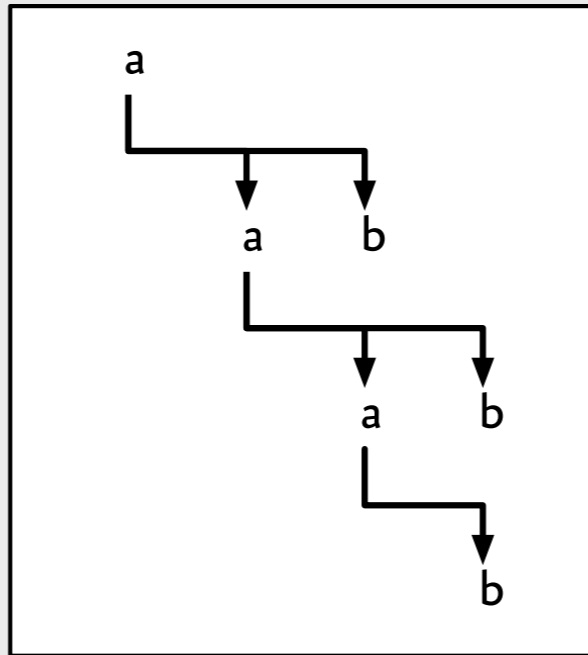
- \* Gaifman proved that lexicalised context-free grammars and projective dependency grammars are equivalent.
- \* **Context-free dependency language:**  
image of a context-free set of trees  
under projective closure

$S \rightarrow a S B$

$S \rightarrow a B$

$B \rightarrow b$

**CF grammar**



**CF tree set**

local order

$$v <_u w$$

---

$$v < w$$

$Suv$

$$v <_u w$$

$Suw$

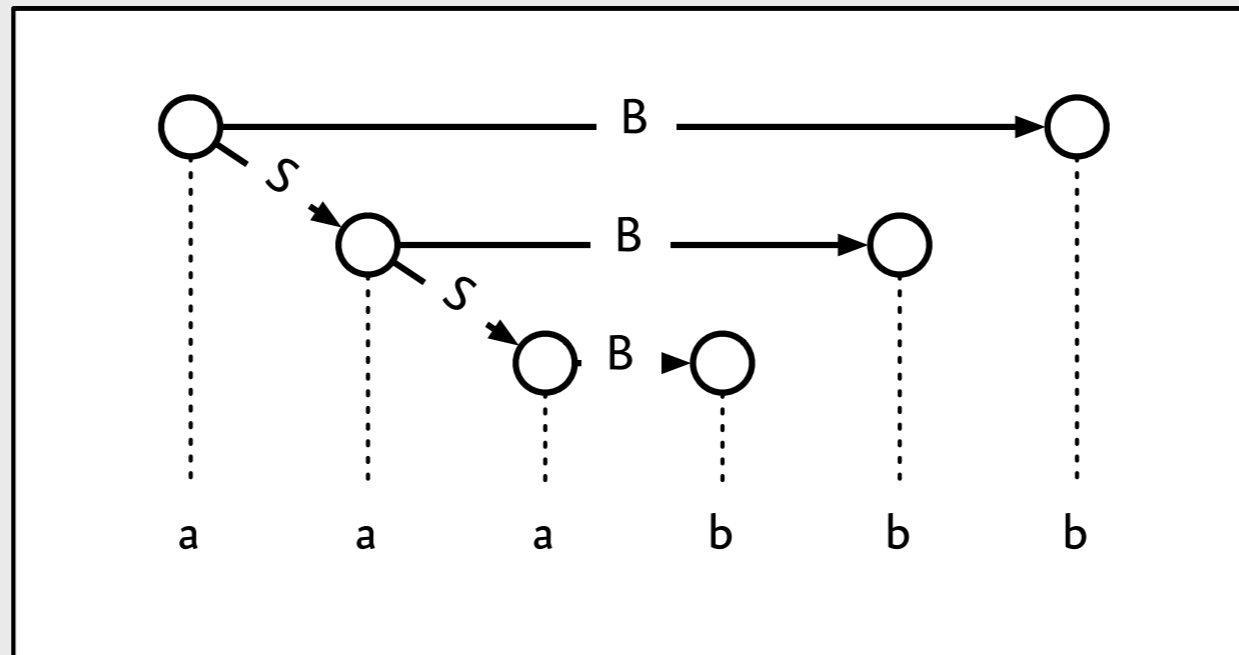
global order

---

$$S^+v < S^+w$$

# Projective closure





**CF dependency  
language**

# Dissecting CF languages

- \* an underlying set of unordered trees:  
**forest structure**
- \* global linearisation constraints:  
**projectivity, non-projectivity**
- \* local linearisation constraints:  
**grammar-specific**

# MCS languages

- \* **mildly context-sensitive languages**
  - \* extend context-free languages
  - \* limited form of non-projectivity
- \* **mildly context-sensitive formalisms**
  - \* TAG, CCG, Minimalist Grammar
  - \* corpora and practical parsers exist

# Properties of MCSL

- \* **limited crossing dependencies:**  
crossed-serial dependencies in Dutch
- \* **constant growth property:**  
the size progression of the language  
is bounded by a constant
- \* **recognisable in polynomial time**

# The plan

- \* Take the sets of (unordered) trees specified by context-free grammars.
- \* Define a class of deterministic automata that transform those trees into dependency structures.
- \* Prove that the resulting output languages are mildly context-sensitive.

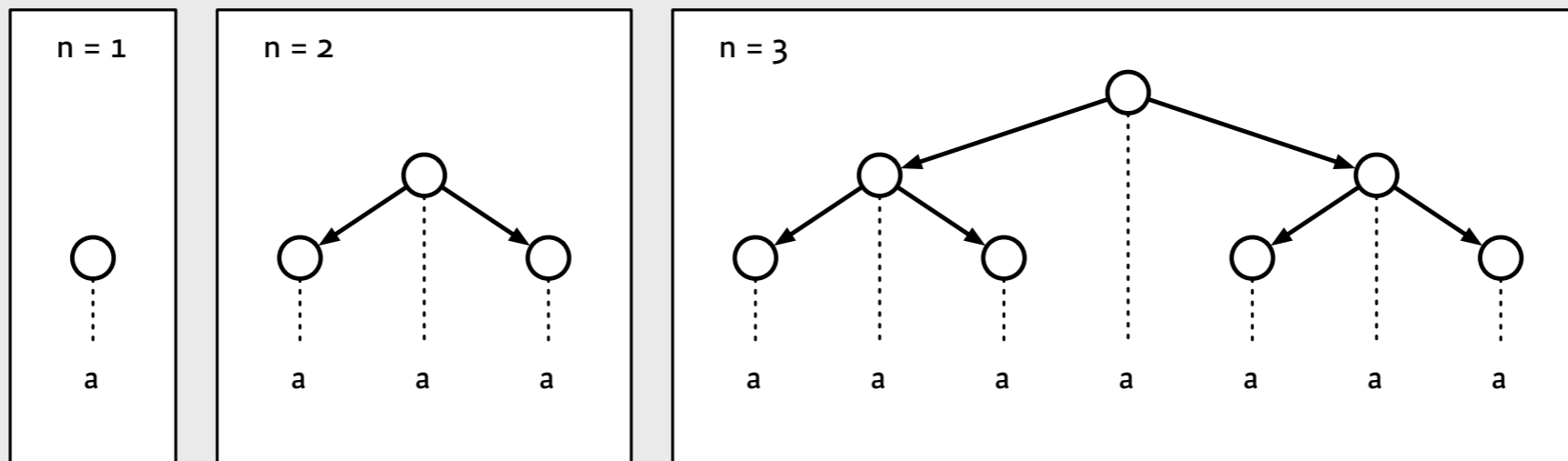
# Semilinearity

- \* Semilinearity is a property that implies the constant growth property.
- \* It may be too strong a constraint to be put on natural language.
- \* A language is **semilinear**, if its Parikh image can be decomposed into a finite union of linear sets.

# Parikh images

- \* **Parikh vector for a string:**  
function that maps terminal symbols to their numbers of occurrences
- \* **Parikh image of a language:**  
set of Parikh vectors  
for the strings in that language

$$\{a^{2^n - 1} \mid n \geq 1\}$$



# Semilinearity



# Tree linearisers

- \* **ingredients:** context free grammar + specification of linearisation
- \* **output:** a set of dependency structures (labelled drawings)
- \* specialisation of deterministic tree-walking transducers (Weir 1992)

# Tree linearisers

- \* finite set of **context-free rules**
- \* finite set of **states**
- \* finite set of **actions**: *up, down n, mark*
- \* **transition function**:  
given a current rule and state,  
what is the action to take,  
and what is the new state?

deterministic

$S \rightarrow a S B$

$S \rightarrow a B$

$B \rightarrow b$

**CF grammar**

$S \rightarrow a S B$

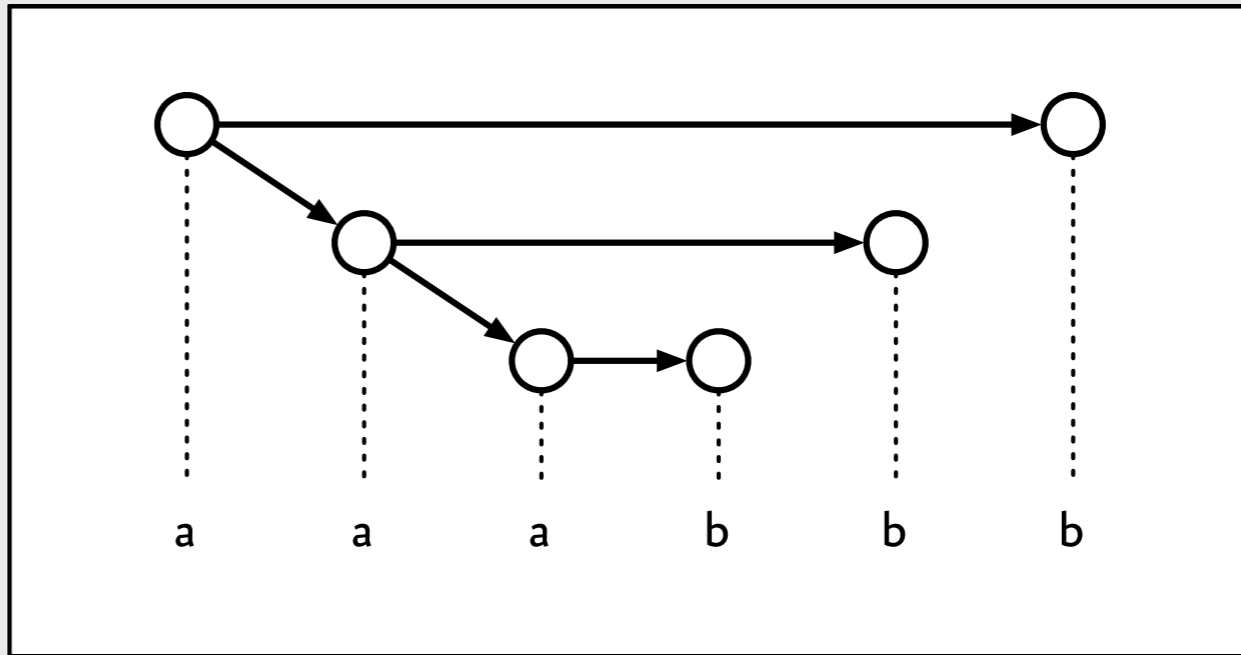
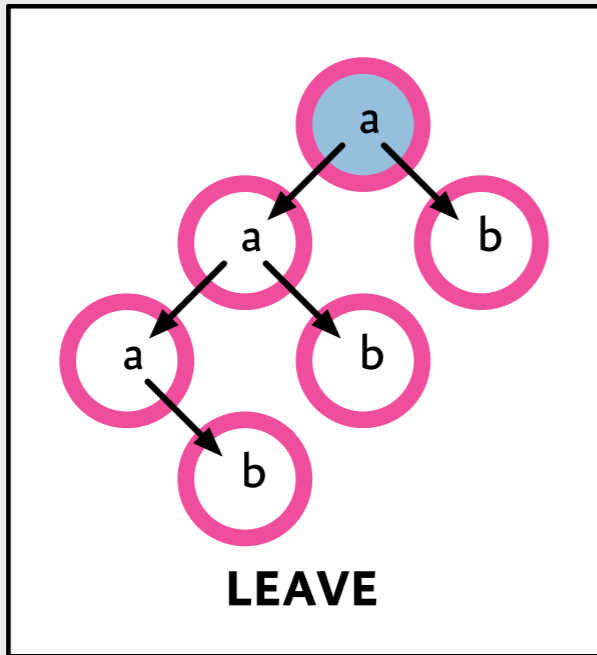
entered  $\rightarrow$  (**MARK**, marked)

marked  $\rightarrow$  (**ENTER S**, entered)

left S  $\rightarrow$  (**ENTER B**, entered)

left B  $\rightarrow$  (**LEAVE**, left S)

# Lineariser 1



# Lineariser 1

# Beyond context-freeness

- \* **reentrancies**: allow a subtree to be entered and left more than once
- \* number of times that this happens is called the **crossing number**
- \* extension introduces a finite number of additional states

$S \rightarrow a S B$

entered 1  $\rightarrow$  (**MARK**, marked)

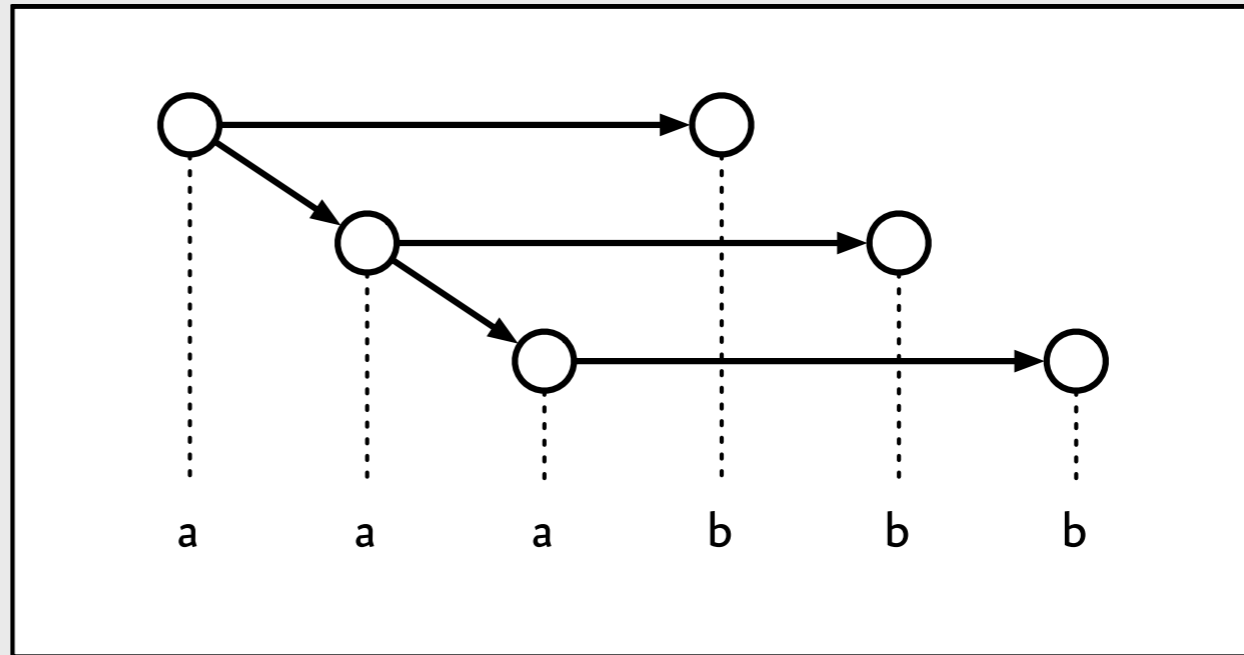
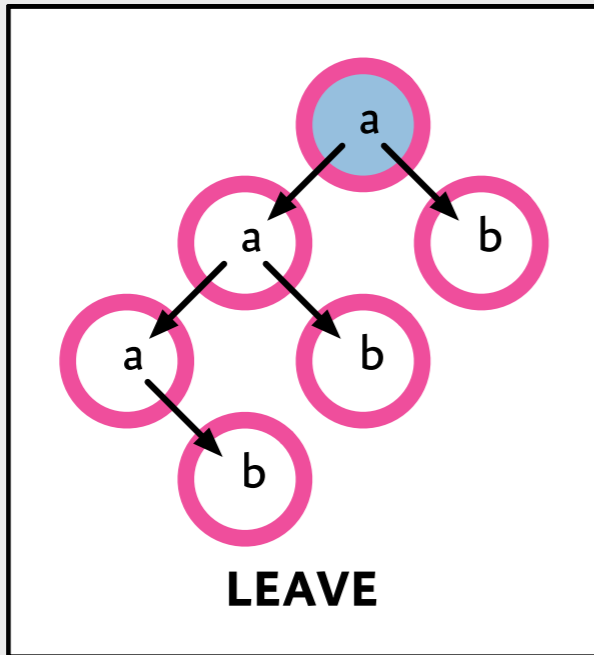
marked  $\rightarrow$  (**ENTER S**, entered 1)

left S 1  $\rightarrow$  (**LEAVE**, left S 1)

entered 2  $\rightarrow$  (**ENTER B**, entered)

left B  $\rightarrow$  (**LEAVE**, left S 2)

# Lineariser 2



# Lineariser 2



# DTL – Finite gap degree

- \* Each subtree can be visited at most a finite number of times.
- \* Therefore, the yield of each node can be split into at most a finite number of convex blocks, with a finite number of gaps.

# DTL – Semi-linearity

- \* **Parikh's Theorem:**

A language is semi-linear if and only if its Parikh image is the Parikh image of a context-free language.

- \* Deterministic tree linearisers merely order the nodes of a context-free set of trees.

# Summary

- \* **basic idea:**

characterise mildly context-sensitive  
dependency languages  
by different tree linearisation regimes

- \* **still to be done:**

show that the output languages  
of tree linearisers  
can be parsed in polynomial time

# A note on grammars

- \* I have taken a completely **extensional approach** to dependency languages.
- \* Ultimately, I also want to be able to explain what a mildly context-sensitive **dependency grammar** is.
- \* I have taken first steps into this direction (Grabowski et al., 2005).

# Structure

- \* Introduction
- \* Dependency structures
- \* Dependency languages
- \* **Complexity**
- \* Conclusion & Future Work

**Complexity**

# Fundamental results

- \* **Eisner 1996:**

Projective dependency grammars can be parsed in time  $O(n^3)$ .

- \* **Neuhaus & Bröker 1997:**

The general word problem for unrestricted dependency grammars is NP-complete.

# Contribution

- \* **general parsing schema**  
for gap-restricted  
dependency languages
- \* **parsing schema:**  
abstract specification of a parsing  
algorithm as an inference system  
(Sikkel 1997)

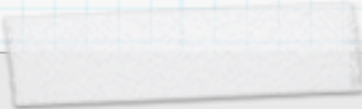


span

$S : \langle I, \Omega \rangle$

type

**Parse items**


$$\frac{\langle I, \Omega \rangle \in Lex(w_i)}{\{i\} : \langle I, \Omega \rangle}$$

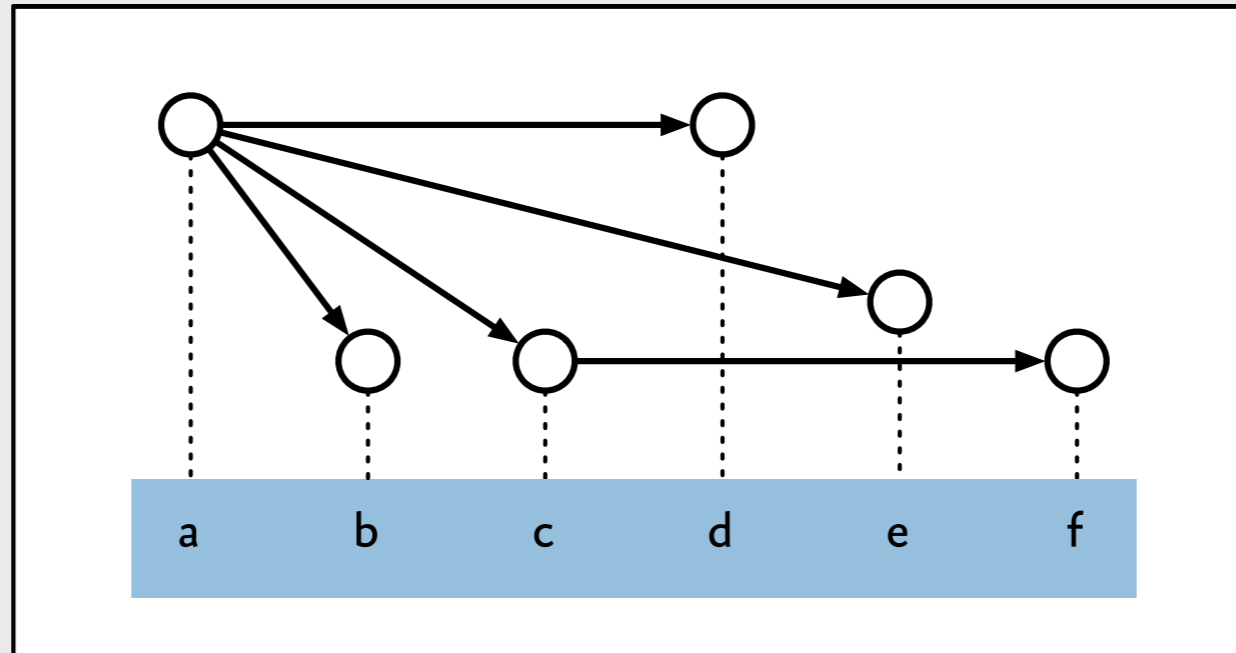
**The LOOKUP rule**

$$\begin{array}{l} s_1 : \langle I_1, \Omega + I_2 \rangle \qquad s_2 : \langle I_2, \emptyset \rangle \\ \hline s_1 \oplus s_2 : \langle I_1, \Omega \rangle \end{array}$$

**The PLUG rule**

$$\begin{array}{c} s_1 : \langle I_1, \emptyset \rangle \quad \cdots \quad s_m : \langle I_m, \emptyset \rangle \\ \hline s_1 \oplus \cdots \oplus s_m : \langle I_1 + \cdots + I_m, \emptyset \rangle \end{array}$$

**The GROUP rule**



# Example

# Results

- \* The parsing schema proves that **arbitrary gap-restricted drawings** can be parsed in polynomial time.
- \* In the special case of **well-nested gap-restricted drawings** a binary group rule suffices, independently of the gap degree.

$$g^{m+2}$$

$$3(g+1)$$

# **Conclusion & Future Work**

# Summary

- \* I have proposed a general framework in which existing DG formalisms can be studied and compared.
- \* I have presented evidence that this framework will provide the notion of mildly context-sensitive dependency grammars.



# Future work (1)

- \* **formal aspects**

- \* dependency *grammars*

- \* embedding more formalisms

- \* **processing issues**

- \* general gap-restricted drawings

- \* issues related to lexicalisation

# Future work (2)

- \* **linguistic relevance**
  - \* study more phenomena
  - \* corpus study on non-projectivity
- \* **implementation**
  - \* using constraint programming
  - \* using dynamic programming

# Polynomial DGs

- \* There are linguistic phenomena that are beyond the expressive power of mildly context-sensitive languages.
- \* It might be interesting to study dependency correspondents of more powerful grammar formalisms, such as Literal Movement Grammar.

**Thank you  
for listening!**

# Backup Slides

# Affiliations

- \* Programming Systems Lab  
Department of Computer Science
- \* Sonderforschungsbereich 378  
Deutsche Forschungsgemeinschaft
- \* International Post-Graduate College  
Language Technology  
and Cognitive Systems

# Collaborators

- \* **Manuel Bodirsky**  
does research in constraint solving at  
the Humboldt-Universität zu Berlin
- \* **Robert Grabowski and  
Mathias Mühl**  
are diploma students at  
Saarland University, Saarbrücken

# Labelled drawings

- \* **Labelled drawings** are drawings equipped with two labelling functions.
- \* **Node labels** correspond to terminal symbols in LCFG.
- \* **Edge labels** correspond to non-terminal symbols.



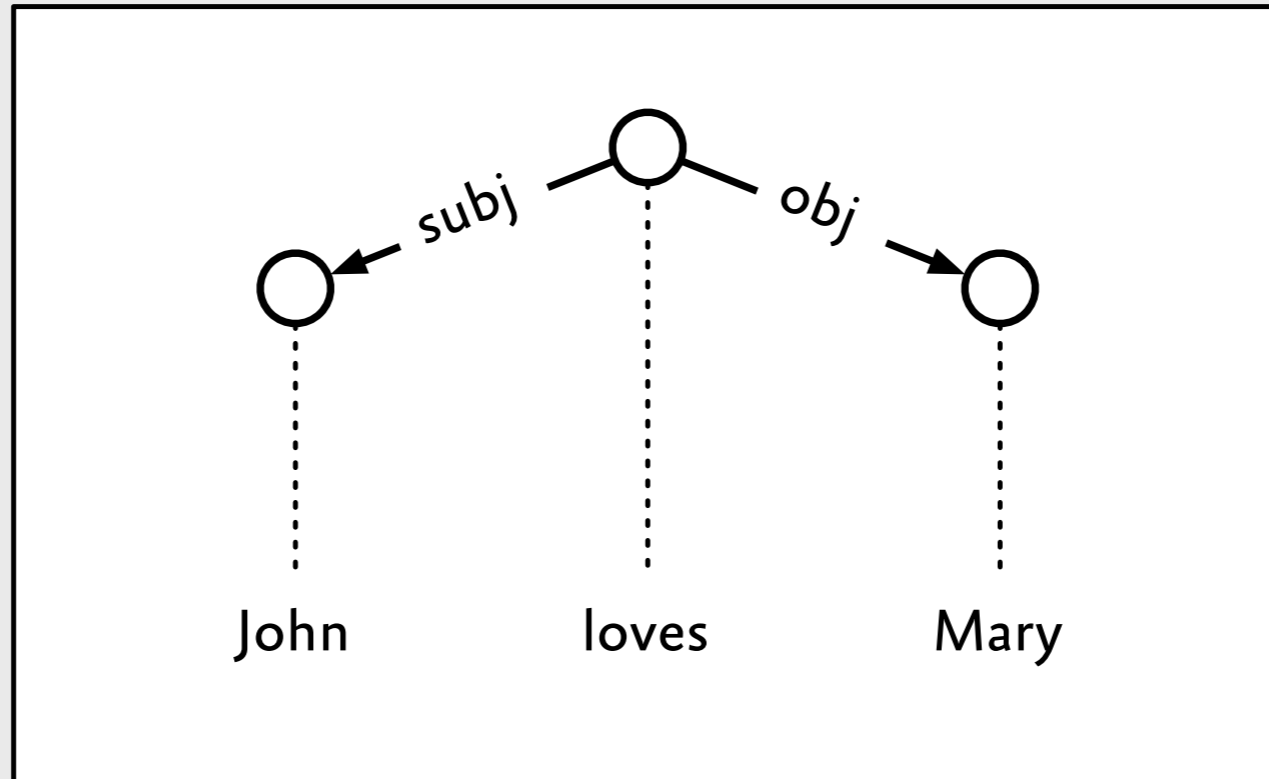


sentence → subj loves obj

subj → John

obj → Mary

# Labelled drawings



**Labelled drawings**

# Properties

- \* Each context-free dependency language is a subset of the set of **projective** dependency structures.
- \* Context-free dependency languages are **closed under (consistent) permutation of subtrees**.